Econ 205A: Problem Set #3

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The following problems are due in class on Wednesday, October 14, 2009. Please turn in one assignment per group

1 N Person Economy– Identical Preferences

Consider the growth model for N identical consumers instead of the single consumer that we discussed in class. Assume that each consumer is given the same initial endowment k_0. That is $k_{i0} = k_0$ for all i.

1. Define and Arrow-Debreu equilibrium for the N person economy.
2. Suppose $\{c_t^*\}_{t=0}^{\infty}$, $\{k_t^*\}_{t=0}^{\infty}$, $\{h_t^*\}_{t=0}^{\infty}$, $\{i_t^*\}_{t=0}^{\infty}$, $\{y_t^*\}_{t=0}^{\infty}$, $\{p_t^*\}_{t=0}^{\infty}$, $\{w_t^*\}_{t=0}^{\infty}$, $\{r_t^*\}_{t=0}^{\infty}$ is the unique equilibrium for the singe consumer economy discussed in class. Show that there is also a unique equilibrium for the N person economy and that in this equilibrium prices are also $\{p_t^*\}_{t=0}^{\infty}$, $\{w_t^*\}_{t=0}^{\infty}$, $\{r_t^*\}_{t=0}^{\infty}$ and that each consumer makes the choices $\{c_t^*\}_{t=0}^{\infty}$, $\{k_t^*\}_{t=0}^{\infty}$, $\{h_t^*\}_{t=0}^{\infty}$, $\{i_t^*\}_{t=0}^{\infty}$.

2 N Person Economy– Non-Identical Preferences

Consider an N person economy where consumers can have different preferences. In particular, suppose that consumer i has preferences given by:

$$\sum_{t=0}^{\infty} \beta^t u_i(c_{it})$$

with k_{i0} given and c_{it} is consumption for agent i in period t.

1. Define and AD equilibrium for this economy.

2. Define a steady state equilibrium for this economy. Hint: you should allow for different levels of capital across individuals but require that each individual's capital remains constant.

3. Show that in steady state the aggregate capital stock K^* must satisfy:

$$f'\left(\frac{K^*}{N}\right) = \frac{1}{\beta} - (1-\delta)$$

4. Consider a social planning problem for this economy where the social planner solves:

$$\max \sum_{i=1}^{N} \alpha_i \sum_{t=0}^{\infty} \beta^t u_i(c_{it})$$

   Where α_i is the social planner's weight associated with agent i. We assume $0 < \alpha_i < 1$ and $\sum_{i=1}^{N} \alpha_i = 1$. Show that the unique aggregate steady state capital stock must satisfy:

$$f'\left(\frac{K^*}{N}\right) = \frac{1}{\beta} - (1-\delta)$$

   independently of the values of the planner's weights.

3 AD vs. SoM II

Consider an endowment economy with two agents. Each agent has identical preferences given by:

$$\sum_{t=0}^{\infty} \beta^t u_i(c_t)$$

Endowments of each agent (ω_1, ω_2) are given by:

$$\omega_i = (\omega_{i1}, \omega_{i2}, \dots)$$

1. Define and AD equilibrium.

2. Define a SoM equilibrium.

3. Suppose that the period utility function is given by $u(c) = \log c$[1]. The endowment of each agent is given by:

$$\omega_1 = (2, 1, 2, 1, 2, 1, \dots)$$
$$\omega_2 = (1, 2, 1, 2, 1, 2, \dots)$$

   Solve for an AD and a SoM equilibrium.

[1] Whenever I write log I mean the natural logarithm, not base 10. The natural logarithm is useful, base 10 is in general not for purposes in economics.

4 Corner Solution!

Consider the growth model in class and assume that the period utility function is given by $u(c) = c$. Show that the social planner's problem has the property that the agent will move to steady state as quickly as feasible. That is the agent will set $k_1 = k^*$ as long as $f(k_0) + (1-\delta)k_0 - k^* > 0$. Otherwise the agent will set $c_0 = 0$ and set $k_2 = k^*$ as long as ...

5 Stability of the Growth Model

Reformulating the standard growth model in class in state control formulation we get the following Euler equation:

$$u'(f(k) + (1-\delta)k - g(k)) = \beta u'(f(g(k)) + (1-\delta)g(k) - g(g(k))) \left[f'(g(k)) + (1-\delta)\right]$$

Where $k' = g(k)$ is the optimal policy function for capital in the next period. In such a dynamic system, stability requires that $0 < g'(k) < 1$. This problem asks you to show that the neoclassical growth model is stable.

1. Totally differentiate the above equation with respect to k. Rearrange terms to show that you have a quadratic function in $g'(k)$.
2. Impose the steady state conditions to the above equation to write all expressions in terms of only k^*. Note that in this notation steady state requires: $k = g(k) = g(g(k))$.
3. Assume the following functional forms: $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ and $f(k) = Ak^{\alpha}$. Again simplify the quadratic expression.
4. Letting $\beta = 0.96$, $A = 1$, $\gamma = 0.5$, and $\alpha = 0.33$, find the stable solution for $g'(k)$ at the steady state.