Econ 205A: Problem Set #7

Professor Aspen Gorry

November 13, 2009

The following problems are due by 5:00pm on Friday, November 20, 2009. Please turn in one assignment per group.

1 Durable Good Consumption

This question asks you to write out the Bellman equation for each specification of the growth model that contains a durable good. In each case, preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, d_t)$$

and the individual is endowed with one unit of time that is supplied as labor in each period. For each specification of technology, write down the Bellman equation that could be used to solve the problem (you do not have to do any analysis).

1. Putty-putty: Output can be used as durables, capital, and consumption. Moreover, capital and durables can be turned back into output in any period. Therefore, feasibility requires:

$$c_t + d_{t+1} + k_{t+1} \leq F(k_t, h_t) + (1 - \delta^d)d_t + (1 - \delta^k)k_t$$
$$k_t \geq 0$$
$$d_t \geq 0$$
$$c_t \geq 0$$

2. Putty-clay: Output can be used as durables, capital, and consumption, however once in place durables and capital can no longer be turned into output. Feasibility now requires:

$$c_t + i_t^d + i_t^k \leq f(k_t, h_t)$$
$$k_{t+1} = (1 - \delta^k)k_t + i_t^k$$

$$d_{t+1} = (1-\delta^d)d_t + i_t^d$$
$$c_t \geq 0$$
$$i_t^d \geq 0$$
$$i_t^k \geq 0$$

3. Clay-clay: Now each good is produced with its own production function and capital and durables cannot be turned into output. However, capital can be reallocated between the three uses in each period. Feasibility requires:

$$c_t = f^c(k_{ct}, h_{ct})$$
$$i_t^d = f^d(k_{dt}, h_{dt})$$
$$i_t^k = f^c(k_{kt}, h_{kt})$$
$$d_{t+1} = (1-\delta^d)d_t + i_t^d$$
$$k_{t+1} = (1-\delta^k)k_t + i_t^k$$
$$h_{ct} + h_{dt} + h_{kt} = 1$$
$$k_{ct} + k_{dt} + k_{kt} = k_t$$
$$c_t \geq 0$$
$$i_t^d \geq 0$$
$$i_t^k \geq 0$$

2 Fixed Capital Investment

In the clay-clay example above assume instead that once capital is installed for a particular use that it is not able to be redeployed for another use. Write down the feasibility conditions that correspond to this assumption. Finally, write down a Bellman equation that could be used to solve this problem.

3 Time to Build

In the growth model discussed in class investments in one period become productive in the next period. This implies that the time to build is one period. Now we will consider a two period time to build model. Assume that in order to increase the capital stock in period $t+2$ by 1 unit it requires an initial investment of $\frac{1}{2}$ unit in period t and an initial $\frac{1}{2}$ unit in period $t+1$. In particular, let i_{1t} be investment in new projects in period t, and let i_{2t} be investment in ongoing projects in period t.

Let s_t be the stock of unfinished projects in period t that were begun in $t-1$ ($s_t = 2i_{1t-1}$). Then the law of motion for the (productive) stock of capital is:

$$k_{t+1} = (1-\delta)k_t + \min(s_t, 2i_{2t})$$

Preferences are standard:

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

The production function is:

$$F(k_t, h_t)$$

1. Write down a Bellman equation for the social planner's problem.
2. Suppose we generalize to 4 periods of time to build with $\frac{1}{4}$ unit of investment in each of 4 periods to produce an additional unit of capital in the 5th period. Formulate the Bellman equation in this case.

4 Human Capital Accumulation

This question provides a model of human capital accumulation. Preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

In each period, agents are endowed with one unit of time to split between working and learning. Let h_{1t} be time spent working in period t, and let h_{2t} be time devoted to studying in period t. The worker's stock of human capital in period t is denoted by h_t. It evolves according to:

$$h_{t+1} = (1-\delta)h_t + f(h_{2t})$$

Output in period t is given by:

$$y_t = h_t h_{1t}$$

Initial human capital is h_0. There is no borrowing or lending. Write down the Bellman equation corresponding to the worker's optimization problem.