Econ 205A: Problem Set #8

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The following problems are due in class on December 4, 2009. Please turn in one assignment per group[1].

1 Oil Prices Revisited

Consider an economy that uses oil as an input to production. The price of oil p_t is set by a cartel and is stochastic. In particular, the price is i.i.d. over time with density $q(p)$ on a closed interval $[1, \bar{p}]$, with $\bar{p} > 1$.

Preferences of the representative consumer are given by, $E_0\left[\sum_{t=0}^{\infty} \beta^t u(c_t, 1 - h_t)\right]$, where c_t is consumption, $1 - h_t$ is leisure, and u is continuously differentiable, strictly increasing, and strictly concave, and $0 < \beta < 1$.

A representative firm produces with the following technology:

$$y_t = A k_t^{\alpha} e_t^{\gamma} h_t^{1-\alpha-\gamma}$$

where k_t, e_t, and h_t are capital, energy, and labor inputs to production. A, α, $\gamma > 0$ and $\alpha + \gamma < 1$. The depreciation rate for capital is 100%, $(\delta = 1)$, so the resource constraint for the economy is:

$$y_t = c_t + k_{t+1} + p_t e_t$$

1. Write down the Bellman Equation for the social planner's problem.
2. Write down the first order conditions and the Euler equation (you should have two static first order conditions, the Euler equation, and the law of motion for capital from the Bellman Equation).

[1]Problems based on questions from Nancy Stokey.

3. For the rest of the question let the utility function have the form:

$$u(c, 1-h) = \log(c) + b\log(1-h)$$

with $b > 0$.

Characterize the optimal savings rate, labor input, and energy input as clearly as possible. First, find an equation for optimal energy consumption in terms of k, h, and p. Then write net output $\hat{y} = y - pe$ in terms of k, h, and p. Eliminate e from the other first order conditions. Then write consumption and capital next period in terms of the savings rate s: $c = (1-s)\hat{y}$, $k' = s\hat{y}$. Characterize the optimal savings rate s^*. How does it depend on (k, p)? Characterize the optimal labor input h. How does it depend on (k, p)?

4. Write a linear stochastic difference equation for the log of teh capital stock, $\log k_t$, where the forcing term is the log of the oil price, $\log p_t$.

5. The capital stock fluctuates in the long run, but remains inside a closed interval. What are the bounds of this interval?

2 Labor Search

A worker can freely switch between two jobs, A and B. The wage at either job is describes by a job-specific n-state Markov chain based on the last wage earned by the worker in that particular job. Each period the worker works at either job A or B. At the end of the period, before observing next period's wage on either job, she chooses which job to go to. We use lower case letters ($i, j = 1, 2, 3, \ldots, n$) to denote states for job A and upper case letters, ($I, J = 1, 2, 3, \ldots, n$) for job B.

Let $w_A(i)$ be the wage on job $A$when state i occurs and $w_B(I)$ be the wage on job B when state I occurs. Let $A = [A_{ij}]$ be the matrix of one step transition probabilities between the states on job A, and let $B = [B_{IJ}]$ be the matrix for job B. If the workers leaves a job and later returns to it, her state on that job is the last wage she received at it. That is, the wage on a job does not change while the worker is working at the other job.

The worker's objective function is to maximize the expected discounted value of her lifetime earnings, $E_0 \sum_{t=0}^{\infty} \beta^t y_t$, where $0 < \beta < 1$ is the discount factor, and where y_t is her wage from whichever job she is working at in period t.

1. Suppose that $w_A(i)$ was the last wage the worker received on job A and $w_B(I)$ the last wage on job B. Write the Bellman equation for the worker.

2. Suppose that the worker is just entering the labor force. The first time she works at job A, the probability distribution for her initial wage is $\pi_A = (\pi_{A1}, \ldots, \pi_{An})$. Similarly, the probability distribution for her initial wage on job B is $\pi_B = (\pi_{B1}, \ldots, \pi_{Bn})$. Formulate the decision problem for a new worker, who most decide which job to take initially. [Hint: Let $v_A(i)$ be the expected discounted value of lifetime earnings for a worker who was last in state i on job A and has never worked on job B. Devine $v_B(i)$ symmetrically].