October 7th 2009 – Lecture Three

Competitive Equilibrium, in the Arrow-Debreu Neoclassical Growth Model

It's a list of sequences $\{c_t\}_{t=0}^{\infty}, \{k_t^c\}_{t=0}^{\infty}, \{h_t^c\}_{t=0}^{\infty}, \{k_t^f\}_{t=0}^{\infty}, \{h_t^f\}_{t=0}^{\infty}, \{i_t\}_{t=0}^{\infty}, \{y_t\}_{t=0}^{\infty}, \{p_t\}_{t=0}^{\infty}, \{w_t\}_{t=0}^{\infty}, \{r_t\}_{t=0}^{\infty}$,

(1) given p, w, r & k_0

Consumer Maximization Problem

They control c_t, k_t^c, i_t, h_t^c

$$(2)\ \max \sum_{t=0}^{\infty} \beta^t\ u(c_t)$$

$$(3)\ s.t.\ \sum_{t=0}^{\infty}(p_t c_t + p_t i_t) = \sum_{t=0}^{\infty}(w_t h_t^c + r_t k_t^c)$$

Technically there is an ≤ sign between the two summations.
But that will always hold with equality.

$$(4)\ s.t.\ k_{t+1} = (1-\delta)k_t + i_t$$
$$(5)\ c_t \geq 0 \quad - \quad (6)\ 0 \leq h_t < 1$$

You may notice with our summations, that we are doing summations of infinite sums, meaning $\infty = \infty$. That is not a well posed question? We can restrict $p_t, r_t\ and\ w_t$ (the prices) to ensure this question is valid.

Producers Maximization Problem

They control k_t^f, h_t^f, y_t

$$(7)\ \max \sum_{t=0}^{\infty}\left(p_t y_t - r_t k_t^f - w_t h_t^f\right)$$
$$(8)\ s.t.\ y_t = f\left(k_t^f, h_t^f\right)$$
$$(9)\ k_t^f \geq 0 \quad - \quad (10)\ h_t^f \geq 0$$

Markets Clear

$$(11)\ k_t^c = k_t^f = k_t$$
$$(12)\ h_t^c = h_t^f = h_t$$
$$(13)\ y_t = c_t + i_t \quad \forall t$$

To solve CE,

$$h_t = 1, \quad \& \quad i = k_{t+1} - (1-\delta)k_t$$

The Consumer Problem

Solve
$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$
$$s.t. \sum_{t=0}^{\infty} p_t(c_t + k_{t+1}) = \sum_{t=0}^{\infty}(r_t k_t + w_t(1) + p_t(1-\delta)k_t)$$

FOCs

c_t: (1) $\beta^t u'(c_t) = \lambda p_p$

k_t: (2) $\lambda p_{t-1} = \lambda\left(r + p_t(1-\delta)\right)$

or just $p_{t-1} = r + p_t(1-\delta)$

$$\max p_t F(k_t, h_t) - r_t k_t - w_t h_t$$

FOCs

k_t: (3) $p_t F_k = r_t$
h_t: (4) $p_t F_h = w_t$

Eliminate $p_t, r_t, w_t \text{ and } \lambda$

$$(1)\ \frac{\beta^t u'(c_t)}{\beta^{t-1} u'(c_{t-1})} = \frac{\lambda p_t}{\lambda p_{t-1}}$$

detrending

$$(i)\ \frac{\beta u'(c_t)}{u'(c_{t-1})} = \frac{p_t}{p_{t-1}}$$

Now switching over the equation two

$$(2)\ \frac{p_{t-1}}{p_t} = \frac{r_t}{p_t} + (1-\delta)$$

Moving to equation three

$$(3)\ \frac{r_t}{p_t} = F_k$$

Now take inverse of (i) and plug it into (3)

$$\frac{u'(c_{t-1})}{\beta u'(c_t)} = F_k + (1-\delta)$$

Euler's Equation
defined here (pre-steady state)

$$\frac{u'(c_{t-1})}{\beta u'(c_t)} = F_k + (1-\delta)$$

It relates consumption today to consumption tomorrow – which depends on discounting, depreciations and the wage rate

Does the above look familiar?

$$F_k = \frac{1}{\beta} - (1-\delta)$$

Thus the prices in the SS

$$\frac{\beta u'(c^*)}{u'(c^*)} = \frac{p_t}{p_{t+1}} \rightarrow \frac{p_{t-1}}{p_t} = \frac{1}{\beta}$$

■ $p_t = \beta p_{t-1}$ $\quad WOW!, \quad noteworthy$

Now think about the other prices

$$F_k = \frac{r_t}{p_t} = \frac{r_{t-1}}{p_{t-1}}$$

$$\frac{r_{t-1}}{r_t} = \frac{p_{t-1}}{p_t} = \frac{1}{\beta} \rightarrow$$

$r_t = \beta r_{t-1}$ & $w_t = \beta w_{t-1}$