October 21st 2009 – Lecture Eight

Gorry added a solution to WEBCT to Problem Set 3 – final question.

Today, we'll be adding more economics to our toolboxes of discrete and continuous time problems.

Labour Market – tradeoffs

1) Consumption Today vs Future Consumption.

That is, given some output, we must choose an allocation over time.

2) Consumption vs Leisure

Given capital, how much should we produce? Implicitly, how much should we work? (and relax…)

So far we've worked with one model

A Simple Model of Consumption vs Leisure

Preferences

$$u(c, 1-h)$$

$u: C^2, strictly\ concave, all\ Inada\ conditions\ hold$

Where c is consumption,
h is amount worked,
1 unit of time a day
1-h is total leisure

Technology

$$y = Ah$$

Endowments

1 unit of time
ω units of output

Social Planner Problem (no prices)

$$\max_{c,h} u(c, 1-h)$$

$$s.t. \quad c = \omega + Ah$$

Implies that, solve the problem, max over h by substituting out c.

$$\max_{h}\ u(\omega + Ah, 1-h)$$

FOC

$$u_1(\cdot)A - u_2(\cdot) = 0$$

$$u_1(c, 1-h)A = u_2(c, 1-h)$$

$$A = \frac{u_2}{u_1}$$

Where,
$u_2(c, 1-h)/u_1(c, 1-h)$ is the Marginal Rate of Substitution between consumption and leisure. That is, how people feel about consumption vs leisure.

A is the marginal rate of technical substitution (marginal rate of transformation). How a change in the input produces a change in the output. AKA "what is technically possible".

With this, we can ask (and answer), if you give up leisure how much consumption will you get? You can ask "If A changes, how will that affect c and (1-h)? You can change endowment and answer similar questions.

Comparative Statics

Suppose that

1) A increases – more productive. There is an income effect, you're wealthier. You'd expect to increase consumption. However, there is a substitution effect too – perhaps because you're so much more productive (think large permanent increase in income) you decide to work less. So the total effect is less clear.
2) ω increases – you have more endowed. There is just an income effect.

The income effect could affect both consumption and leisure too – you want to consume more and you can take some leisure.

We can answer this with Comparative Statics

$$\frac{u_2(\omega + Ah, 1-h)}{u_1(\omega + Ah, 1-h)} = A$$

Say that $u(c, 1-h) = \frac{c^\sigma}{\sigma} + \frac{\alpha(1-h)^\gamma}{\gamma}$

Then $u_1(\omega + Ah, 1-h) = c^{\sigma-1}$

$$= (\omega + Ah)^{\sigma-1}$$

$and\ u_2(\omega + Ah, 1-h) = \alpha(1-h)^{\gamma-1}$

Thus,

$$\frac{u_2'}{u_1'} = (\alpha(1-h)^{\gamma-1})(\omega + Ah)^{1-\sigma} = A$$

Now take the total differential

$$\frac{d}{d\omega} = -\alpha(\gamma-1)(1-h)^{\gamma-2}(\omega + Ah)^{1-\sigma}\frac{\delta h}{\delta \omega} + (\alpha(1-h)^{\gamma-1})(1-\sigma)(\omega + Ah)^{1-\sigma}\left(1 + A\frac{\delta h}{\delta \omega}\right) = 0$$

Canceling terms – simplifying

$$-(\gamma-1)(\omega + Ah)\frac{\delta h}{\delta \omega} + (1-h)(1-\sigma) + A(1-h)(a-\gamma)\frac{\delta h}{\delta \omega} = 0$$

Solve for $\frac{\delta h}{\delta \omega}$

$$\frac{\delta h}{\delta \omega} = -\frac{(\sigma-1)(1-h)}{(\gamma-1)(\omega + Ah) + (\gamma-1)(1-h)A}$$

$We\ know \quad (1-h) > 0 \qquad (\omega + Ah) > 0 \qquad A > 0$

What about $(\sigma - 1)$ & $(\gamma - 1)$?

Because we have constant elasticity of production function, $0 < \gamma, \sigma < 1$ thus both terms are negative.

$$\frac{\delta h}{\delta \omega} = -\frac{(-)(+)}{(-)(+) + (-)(+)} \Rightarrow \frac{(-)(-)}{(-)} < 0 \qquad -\ very\ likely\ negative$$

Conclusion

Thus $\frac{\delta h}{\delta \omega} < 0$ A increase in endowment, ω will result in less hours worked, h.

Scenarios

- $\omega \uparrow: c \uparrow \,\&\, (1-h) unchanged$
  You get more wealth, and you just want to consume more. Your leisure is unchanged.
- $\omega \uparrow: c\ is\ unchanged \,\&\, (1-h) \uparrow$
  Possibly your endowment goes up and you want more time off, but you don't consume more.
- $\omega \uparrow: c \uparrow \,\&\, (1-h) \uparrow$
  and of course, perhaps both your leisure and consumption rise with added endowment.

(The lecture gets a little complicated) – You actually can pick $\sigma's\ \&\ \gamma's$ such that the signs change around. The effect will depend on if the income effect (how a chg in real income chgs consumption) or the substitution effect (as you have more income, you might prefer leisure or consumption differently) dominates your preferences.

In the real world you can examine shocks to people's endowment to estimate these actual values.

Continuous Time – Competitive Equilibrium

A CE is a list c^*, h^*, y^*, w^* such that, (it's w, not $\omega - omega$ is given in t=0)

1) Taking prices p^*, w^* as given, households choose c^*, h^* that solve

$$\max_{c,h} u(c, 1-h)$$
$$s.t.\ \ p^*c \le w^*h + p^*\omega$$
$$c \ge 0, \quad \omega\ is\ given \quad 0 \le h \le 1$$

2) Taking prices p^*, w^* as given, the firm choose y^*, h^* that solve

$$\max_{y\ h}\ p^*y - w^*h$$
$$s.t.\ \ y = Ah$$
$$h \ge 0$$

3) Markets Clear - the total amount of consumption is equal to total output plus endowment

$$c^* = \omega + y^*$$

The mid-term might look just like this – easy points in setting it up right. He feels that setting this up in a complicated model like this makes solving it very much easier (since you'll have what you're trying to solve for written right up there). You see the terms and know the # of conditions you'll need So learn, understand… memorize, whatever.

Homework might be (midterm might be) –add preferences over leisure to the neoclassical growth model.
Define a competitive equilibrium, solve it.
Solve the social planner's problem of a neoclassical growth model with leisure in it.

Let's solve this

Step one – what's consumption?

$$c = \frac{w^*h + p^*\omega}{p^*}$$

* You get to normalize a price! If there is multiple markets – if you double all prices the allocation is the same (2X $p's$ & $w's$ nothing changes in the economy) You only need one relative price.
This means you can set $p^* = 1$

Solve the consumer's problem

$$\max_{h} u(w^*h + \omega, 1-h)$$

FOC: $u_1'(w^*h + \omega, 1-h)w^* = u_2'(w^*h + \omega, 1-h)$ (continued…..)

FOC: $u_1'(w^*h + \omega, 1-h)w^* = u_2'(w^*h + \omega, 1-h)$

$$\frac{u_2'}{u_1'} = w^*$$

Now solving the firm's problem

$$\max_h Ah - w^* h$$

FOC: $A - w^* = 0$

$$A = w^*$$

Combining the two,

$$\frac{u_2'}{u_1'} = A$$

We get the same equilibrium condition as a solution as we had in the social planner problem.

Not surprising, w^* is the marginal product of labor – which is equal to the marginal rate of transformation A.

Going Back to the Neoclassical Growth Model

Preferences

$$\sum_{t=o}^{\infty} \beta^t \, u(c, 1 - h_t)$$

Technology

$$y_t = F(k_t, h_t)$$

Law of Motion

$$k_{t+1} = (1 - \delta)k_t + i_t$$

Resource Constraint

$$c_t + i_t = y_t$$

Regularity Conditions

$$i_t \geq 0, \;\; c_t \geq 0$$

Endowments

$$k_0 \; is \; given, \;\; 1 \; unit \; of \; time$$

There are two ways we know how to approach this – Social Planner & Competitive Equilibrium

Social Planner's Problem (1st)

$$\max_{k_t,h_t} \sum_{t=0}^{\infty} \beta^t \, u(F(k_t,h_t) + (1-\delta)k_t - k_{t+1}, 1-h_t)$$

FOCs:

k_t:

$$\beta^t u_1(c_t, 1-h_1)[F_k(k_t,h_t) + (1-\delta)] + (-1)\beta^{t-1} \, u_1(c_{t-1}, 1-h_{t-1}) = 0$$

$$u_1{}'(c_{t+1}, 1-h_{t+1}) = \beta u_1'(c_t, 1-h_t)[F_k(k_t,h_t) + (1-\delta)]$$

h_t:

$$\beta^t u_1'(c_t, 1-h_t)[F_h(k_t,h_t)] + (-1)\beta^t u_2'(c_t, 1-h_t) = 0$$

$$u_1'(c_t, 1-h_t)[F_h(k_t,h_t)] = u_2'(c_t, 1-h_t)$$

Thus

$$\frac{u_2'}{u_1'} = F_h(k_t,h_t) \qquad \& \qquad \frac{u_1'(c_{t+1}, 1-h_{t+1})}{\beta u_1'(c_t, 1-h_t)} = F_k(k_t,h_t) + (1-\delta)$$