# October 23rd 2009 - Lecture Nine

<u>Next Lectures Will</u>
Add labor, taxes
talk about econ growth
include dynamic programming
labor search & unemployment

**Types of Problems on the Mid-Term**

In problem sets we've been given two types of problems
– some that are very challenging, that you have to work through and solve and learn something new – things that he thinks we need to know but he didn't cover in lecture.
–Other problems are simple, 'set this up correctly and solve the problem' type queries that you'll see on the mid-term. Basically, make sure you've learning this material.

## Reviewing Problem Set Three - #3

**An Arrow-Debreu (AD) Competitive Equilibrium (CE)** is a list of prices $\{p_t\}_{t=0}^{\infty}$ and allocations $\{c_{it}\}_{t=0}^{\infty}$ such that – taking prices as given, $\{p_t^*\}_{t=0}^{\infty}$, the allocations $\{c_{it}^*\}_{t=0}^{\infty}$ solve for each consumer the following problem,

$$\max_{\{c_{it}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_{it}) \qquad s.t. \sum_{t=0}^{\infty} p_t^* c_{it} = \sum_{t=0}^{\infty} p_t^* \omega_{it} \quad , \quad c_{it} \geq 0 \quad \forall t$$

**Markets Clear Each Period**

$$\sum_{i=1}^{N} c_{it} = \sum_{i=1}^{N} \omega_{it} \quad \forall t$$

**A Sequence of Markets (SoM) Competitive Equilibrium (CE)** is a list of prices, $\{p_t\}_{t=0}^{\infty}, \{q_t\}_{t=0}^{\infty}$ and allocations $\{c_{it}\}_{t=0}^{\infty}, \{b_{it}\}_{t=0}^{\infty}$ such that – taking prices are given $\{p_t^*\}_{t=0}^{\infty}, \{q_t^*\}_{t=0}^{\infty}$, the allocations $\{c_{it}^*\}_{t=0}^{\infty}, \{b_{it}\}_{t=0}^{\infty}$ solve for each consumer the following problem,

$$\max_{\{c_{it}, b_{it}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_{it}) \qquad s.t. \sum_{t=0}^{\infty} p_t^* c_{it} + q_t^* b_{it} = \sum_{t=0}^{\infty} p_t^* w_{it} + q_t^* b_{it-1} \quad , \quad c_{it} \geq 0 \quad \forall t$$

**Markets Clear Each Period**

$$\sum_{i=1}^{N} c_{it} = \sum_{i=1}^{N} \omega_{it} \quad , \quad \sum_{i=1}^{N} b_{it} = 0 \quad \forall t$$

**Solving for Sequence of Markets**

Substitute out consumption, you get to normalize one price

$$\max_{b_{it}} \sum_{t=0}^{\infty} \beta^t \, u(\omega_{it} + b_{it-1} - q^* b_{it})$$

FOC

$$\beta^{t+1} u'(c_{it+1}) = q^* \beta u'(c_{it})$$

Detrending

$$\beta u'(c_{it+1}) = q^* u'(c_{it})$$

Plugging in functional forms $u(c) = \ln(c)$

$$\beta \cdot \frac{1}{c_{it+1}} = \frac{1}{c_{it}} q^*$$

Cross multiply

$$\frac{c_{it}}{c_{it+1}} = \frac{q^*}{\beta} \qquad \textit{this implies that} \quad \frac{c_{1t}}{c_{1t+1}} = \frac{q^*}{\beta} \quad \& \quad \frac{c_{2t}}{c_{2t+1}} = \frac{q^*}{\beta}$$

Adding them together

$$\beta(c_{1t} + c_{2t}) = q^*(c_{1t+1} + c_{2t+1})$$

**Problem Set 4 #2**
**Competitive Equilibrium**

1) Household - Hamiltonian

$$\mathcal{H} = u(c) - v(y) + \lambda \ [rk + w - c]$$

notice that the –v(y) will be ignored

2) $\quad u'(c) = \lambda \ \rightarrow \quad u''(c)\dot{c} = \dot{\lambda}$

3) $\quad \dot{\lambda} = \lambda\rho - r\lambda = 0$

$$\Rightarrow \frac{\dot{\lambda}}{\lambda} = \rho - r$$

Firm $\quad r = f'(k)$

Combining 2) 3) and the firm together,

$$\frac{u''(c)\dot{c}}{u'(c)} = \rho - f'(k)$$

Imposing the SS

$$f'(k^*) = \rho$$

**Social Planner** – substitute y with f(k), the SP is only constrained by what is technologically feasible

$$\mathcal{H} = u(c) - v\big(f(k)\big) + \lambda[f(k) - c]$$

(1) $u'(c) = \lambda$

$\rightarrow \quad u''(c)\dot{c} = \dot{\lambda}$

(2) $\dot{\lambda} = \rho\lambda + v'(y)f'(k) - \lambda f'(k)$

$$\frac{\dot{\lambda}}{\lambda} = \rho - f'(k) + \frac{v'(y)f'(k)}{u'(c)}$$

Combining (1) & (2)

$$\frac{u''(c)\dot{c}}{u'(c)} = \rho - f'(k) + \frac{v'(y)f'(k)}{u'(c)}$$

Conclusion

$$\left(1 - \frac{v'}{u'}\right) f'(k^*) = \rho$$