October 26th 2009 - Lecture Ten

This Lectures Will
Cover answers to Problems Set 4 # 1, discuss Problem Set 3 # 4, and set-up Problem Set 3 #3.
Introduce Government and Taxes to the Neoclassical Growth Model we've been working with. First introducing capital taxation and transfer payments.

Problem Set 4, Problem 1

A Competitive Equilibrium is a list of functions $c(t), k(t), h(t), y(t), x(t), r(t), w(t)$, such that

1) Consumers maximize, taking prices $r(t), w(t)$ as given, choose $c(t), k(t), h(t), x(t)$ that solve,

$$\max \quad \int_0^\infty e^{-\rho t}\left[u\big(c(t)\big) - v\big(h(t)\big)\right]dt$$
$$s.t. \quad c(t) + x(t) = r(t)k(t) + w(t)h(t)$$
$$\dot{k}(t) = x(t) - \delta k(t)$$

"$-v\big(h(t)\big)$" means there is some disutility of labor. Otherwise this is pretty strait forward and otherwise covered in previous lectures.

2) Firms Maximize - Taking prices, $r(t), w(t)$, as given, choose $y(t), h(t), k(t)$ that solve the following equation,

$$\max \quad y(t) - r(t)k(t) - w(t)h(t)$$
$$s.t. \quad y(t) = F\big(k(t), h(t)\big)$$

3) Markets Clear

$$c(t) + x(t) = y(t)$$

The Household Problem – To do our part in this in alleviating the lead shortage, we'll be dropping the $(t)'s$ from our notation

Budget Constraint $\dot{k} = rk + wh - c$

$$\mathcal{H} = u(c) - v(h) + \lambda[rk + wh - c - \delta k]$$

First Order Conditions for HH

$$(c): u'(c) = \lambda$$
$$(h): v'(h) = \lambda$$
$$(\lambda): \dot{\lambda} = \rho\lambda - \lambda r + \delta\lambda$$

Firm's FOCs

$$(k): r = F_1(k, h)$$
$$(h): w = F_2(k, h)$$

Keep in mind that, in addition to following the math to find equations that characterize our solution, we want those equations to make economic sense. Thus,

Equations that Characterize the Steady State

Manipulating (c)

$$\Rightarrow \frac{\dot{\lambda}}{\lambda} = \frac{u''(c)\dot{c}}{u'(c)}$$

$(\boldsymbol{c})\&\,(\boldsymbol{\lambda})$

$$\Rightarrow \frac{u''(c)\dot{c}}{u'(c)} = \rho + \delta - r$$

$(\boldsymbol{h})/(\boldsymbol{c})$

$$\frac{v'(h)}{u'(c)} = w \quad \Rightarrow \quad \frac{v'}{u'} = F_2(k, h)$$

$\frac{v'}{u'} = F_2(k, h)$ – this more clearly describes the tradeoff between the value of consumption (u') & the labor/leisure (v'). w : is the marginal rate of transformation of labor. $\frac{v'}{u'}$ is the marginal substitution between leisure and consumption. So our solution is where these two marginal rates are equation to each other. That's riveting economics there.

One fair question, also, is to ask why not do the manipulation we did to (c) (the left eq up there) to equation (h) instead? Firstly the whole point is to eliminate lambda from these system of equations. We have many lambdas and only one lambda-dot. So we only need to do the (c) manipulation above once to get rid of that lambda-dot. That's why we only want to do that manipulation once, why not $\frac{v''(h)\dot{h}}{v'(h)}$? Well we can argue that the labor supply decision is a static decision made each period – that decision does not carry over into the next like consumption does. Thus we aren't as concerned with an $\dot{h}$ as we are with a $\dot{c}$ $(where\ we\ look\ at\ the\ tradeoff,\ given\ some\ amount\ of\ production\ today, how\ much\ do\ we\ consume\ today\ vs\ consume\ tomorrow?)$

v does enter the economic discussion as we think about the tradeoff of consumption vs leisure –today, $\frac{v'}{u'}$. At what level we set each depends on wages, which depends on characteristics of our production technology $(F_2(k,h))$. That decision is made only day to day, and the consequence of that decision (say, more consumption) is carried over to future in $\frac{u''(c)\dot{c}}{u'(c)}$.

Imposing the Steady State – we are seeking three equations that define c, k & h in the ss. For "find three equations the define the SS" you need to find three equations that are only in terms of those three variables, c, k & h.

$$F_1(k^*,h^*) = \rho + \delta$$

$$\frac{v'(h^*)}{u'(c^*)} = F_2(k^*,h^*)$$

Pulling from our s.t. conditions in the question, $x^* = \delta k^*$

$$c^* + x^* = F(k^*,h^*) \quad - With\ Constant\ Returns\ Production\ function, F(k^*,h^*) = rk^* + wh^* = F_k k^* + F_h h^*$$

From these four equations and four unknowns (we plugged investment x in) you can – (after algebra work) get your answer. 4 eqs and 4 unknowns are messy, but with these you can clean it up pretty quickly.

Interestingly (though not required to solve the problem, but helpful in solving ss problems like these in future more quickly) if $F(k^*,h^*)$ is a constant returns to scale production function (you'll have to prove this or say that you assume it…) it'll define a capital labor ratio in SS. So you can solve this for k/h (as we will in Micro very very soon) That gives you $F_1(k^*,h^*)$ & $F_2(k^*,h^*)$. He's a little vague here on exactly what to do, but he says that "you can have some fun with that – a little trick if I have you functional forms". Right.

Problem Set 3 #3

$$\max \sum_{i=1}^{N} \alpha_1 \sum_{t=o}^{\infty} \beta^t\, u_i(c_{it})$$

Feasibility constraint …

$for\ each\ i, 1, 2, 3, \dots, N$

$$k_{it+1} = (1` - \delta)k_{it} + x_{it}$$

$$\sum_{i=1}^{N} (c_{it} + x_{it}) = F\left(\sum_{i-1}^{N} k_{it}\, , N\right)$$

With this you should be able to set up a Lagranian, FOCs, impose your SS, and find your solution.

Government & Taxes

Substantive Question –

1) Example, why do allocations in an economy change in a particular way? Why are there recessions? What causes output fluctuations over time? What causes the patterns we observe in investment? Etc
2) Policy analysis. How do particular policy changes cause changes in the allocation of goods in the economy? – in this section we'll be looking closely at how changes in tax policy change economic allocations.
3) Why do allocations in different countries differ in the ways they do? Say, why do difference in hours worked differ across borders? Prescott has a paper on this that we'll discuss.

The question we'll introduce to explore all this in a basic way is "How does tax policy change equilibrium allocation?"

We need to make a few assumptions about how tax revenue is used. These aren't necessary all true or as simple as they are here, but these assumptions are needed to deal with the models we've learned.

1) Government could transfer all revenue back to consumers in a lump sum. Typically called transfer payments – think Social Security or Welfare. Our models will suggest that this type of tax won't have any price distortion.
2) Or Government could spend it.

i) Buy a consumption good $\left(u(c+g)\right)$

ii) Buy something that consumer value but can't purchase themselves (roads, financial regulation, public universities, pure scientific research, sewers, clean water, the Marines)

$u(c)+v(g)$ – the + means the good causes utility, a public good

$u(c)-v(g)$ the – sign means disutility. If you're cynical about gov't spending, thinking they waste your money – through it in the ocean.

Thinking About Capital Taxation – with transfer system assumptions

A CE with a tax and transfer system is a list or sequences

$\{c_t^*\}, \{k_t^*\}, \{h_t^*\}, \{i_t^*\}, \{y_t^*\}, \{T_t^*\}$, where $\{T_t^*\}$, is our transfer payments

And prices $\{p_t^*\}, \{r_t^*\}, \{w_t^*\}$,

Such that,

1) Taking prices & taxes as given, $\{T_t^*\}, \{p_t^*\}, \{r_t^*\}, \{w_t^*\}$,

The consumer allocates $\{c_t^*\}, \{k_t^*\}, \{h_t^*\}, \{i_t^*\}$, that solve

$$\max \sum_{t=0}^{\infty} \beta^t\, u(c_t)$$

$$\sum_{t=0}^{\infty} p_t^*(c_t+i_t) = \sum_{t=0}^{\infty} p_t^*\left((1-\tau_k)r_t^* k_t + w_t^* h_t + T_t^*\right)$$

$$k_{t+1} = (1-\delta)k_t + i_t$$

$$c_t \geq 0,\ \ 0 \leq h_t \leq 1,\ \ \ k_0\ given$$

2) Firm's taking prices as given, $\{p_t^*\}, \{r_t^*\}, \{w_t^*\}$, allocate $\{y_t^*\}, \{k_t^*\}, \&\{h_t^*\}$ that solve,

$$\max \sum_{t=0}^{\infty} p_t^*(y_t) - r_t^* k_t - w_t^* h_t$$

$$s.t.\ y_t = F(k_t, h_t)$$

$$k_t \geq 0,\ \ \ h_t \geq 0$$

3) Government Budget Condition – assuming balanced budget (insert necessary joke)

$$T_t^* = \tau_k r_t^* k_t^*$$

4) Market Clearing – just like it normally is

$$c_{it}^* + i_{it}^* = y_{it}^*$$

On market clearing, if government spends money on public goods instead of transfers (that's the second model talked about above) g_t needs to be included here. $c_{it}^* + i_{it}^* + g_t = y_{it}^*$ - This'll come up in a future problem set.

Why are we doing this a Competitive Equilibrium instead of a social planner problem? A tax without prices is pointless. The social planner would solve the problem by internalizing the externality – they'll internalize the distortions of the taxes and you get the same thing out of it. Taxes are like an externality (remember that problem set with pollution? How the social planner internalized the externality while the CE did not). Thus if you want to understand the effect of the taxes you need to deal in prices – with a Competitive Equilibrium.

Characterize the CE

Consumer Budget Constraint – skipping the Lagrangian

$$(\lambda): \sum_{t=0}^{\infty} p_t^*(c_t + k_{t+1} - (1-\delta)k_t) = \sum_{t=0}^{\infty} p_t^*\big((1-\tau_k)r_t^* k_t + w_t^* h_t + T_t^*\big)$$

FOCs

$$(c): \beta^t u'(c_t) = p_t \lambda$$

$$(k): \lambda p_{t-1} = \lambda p_t[(1-\tau_k)r_t + (1-\delta)]$$

Firm's FOCs

$$r_t = F_k(k_t, h_t) = f'(k_t)$$

$$u'(c_t) = \beta u'(c_{t+1})[(1-\tau_k)f'(k_{t+1}) + (1-\delta)]$$

Imposing SS

$$(1-\tau_k)f'(k^*) = \frac{1}{\beta} - (1-\delta) \qquad as\ \tau_k \uparrow, \qquad SS\ k^* \downarrow$$

End of Lecture….