October 30th 2009 - Lecture Eleven

We had the mid-term Oct 28th, the professor has returned our graded tests and today we'll review the correct answer to mid-term question two-of-two – Question one will appear on next week's problem set. Later in the lecture we'll start with the Prescott paper, "Why do Americans Work So Much More than Europeans."

Macro Mid-term 2009 Problem 2 – this would make an excellent study guide to future mid-terms, finals and preliminary exams. Be sure to grab the Problem Set Questions.

We are asked to define a Competitive Equilibrium for the economy.

A CE is a list of functions, $o(t), c(t), h(t), k(t), x(t), y(t)$ and prices $w(t), r(t), q(t)$, such that,

i) Taking prices $w(t), r(t)$ as given, $c(t), h(t), k(t), x(t)$ solve:

$$\int_0^\infty e^{-\rho t}\,[u(c) - v(h)]dt$$

$$s.t.\;\; c + x = wh + rk$$

$$\dot{k} = \dot{x} - \delta k$$

Consumers own all capital. (as a side-note, in the consumer problem you could add the price $p(t)$, but you can just keep the problem more simple)

ii) Taking prices $w(t)\& \, r(t)$ as given, firms choose $o(t), y(t), k(t), h(t)$

As a side note on "taking prices as given" – the firm is acting as if they take prices as given, but it's interesting that the CE is actually determining the prices.

$$\max y(t) - rk - wh - qo \qquad (we've\ dropped\ the\ "\ (t)\ "\ because\ it\ takes\ up\ space)$$

$$s.t.\;\; y(t) = F(k, h, o)$$

iii) Markets Clear

$$y = c + x + q \cdot o$$

Normally we don't want to have prices in the market clearing condition, however the price of oil q is exogenous, given (probably better to say 'taken') by forces beyond the CE.

In the Mid-Term Question, rather distractingly we were given the hint "given the continuous time setup, the natural definition is a sequence of markets equilibrium." In response to this we were told that we have a sequence budget constraint. As he talked about in class last Monday, anytime you have a continuous time setup it looks like a sequence of markets because the budget constraint looks like a period budget constraint rather than the Arrow-Debreu 'total value' budget constraint we've seen more commonly in discrete time problems. So you can think about our market clearing conditions holding in each period – not summed up for all time.

2) The Hamiltonian

$$\mathcal{H} = u(c) - v(h) + \lambda[wh + rk - c - \delta k]$$

Households FOCs

$$c:\;\; u'(c) = \lambda$$

h: $v'(h) = w\lambda$
λ: $\frac{\dot{\lambda}}{\lambda} = \rho - r + \delta$

$$\Rightarrow \frac{\dot{\lambda}}{\lambda} = \frac{u''(c)\dot{c}}{u'(c)} = \rho + \delta - r \qquad , \qquad \frac{v'(h)}{u'(c)} = w$$

3) Firm FOCs

k: $F_k = r$
h: $F_h = w$
o: $F_o = q$

4)

$$\frac{u''(c)}{u'(c)}\dot{c} = \rho + \delta - F_k$$

There is a bunch of economic interpretation here too. But that wasn't part of the mid-term question.

5) A Steady State is a value k^* such that if $k_0 = k^*$, then the function $k(t) = k^*\ \forall t$. (■ memorize this)
Although a lot of us found the four equations asked of us correctly, many people missed points on this problem because we did not state the above incantation for steady state equilibrium, we just said something like "Impose SS, $\dot{k} = 0, \dot{c} = 0$". This was not acceptable. You need the incantation.

The four equations we are asked to find are the following:

$$F_k(k^*, h^*, o^*) = \rho + \delta \qquad F_0(k^*, h^*, o^*) = q$$

$$\frac{v'(h^*)}{u'(c^*)} = F_h(k^*, h^*, o^*) \qquad F(k^*, h^*, o^*) = c^* + \delta k^* + qo^*$$

or the equivolent equation:

$$c^* + x^* = F_h(k^*, h^*, o^*)h^* + F_k(k^*, h^*, o^*)k^*$$

Lecture Notes Continued

Capital Taxes & Welfare Comparisons.

Welfare – effect of policy changes, say, what is the welfare effect of a capital tax?

Consider a static setting with a single good.

Two Economies

Economy 1 – Has an equilibrium c_1

Economy 2 – Has an equilibrium c_2

Let's say that:

$$u(c_1) > u(c_2)$$

This certainly means quantitatively that the consumer is better off in economy #1

Exactly how much better off is a consumer in economy one? Let's think about a few ways to measure this question.

Try One

$u(c_1) - u(c_2)$

No, that won't work. Pointless because utility ordinal only. Number of utils is pointless. $\alpha u(c_1) = \alpha u(c_2)$ denotes the same thing as above.

Try Two

$$\frac{u(c_1) - u(c_2)}{u(c_1)} \quad oh, idiot, but \quad \frac{u(c_1) + \alpha - (u(c_2) + \alpha)}{u(c_1) + \alpha} \text{ is the same thing}$$

No - You can add a constant to a utility function and get the same thing. Once again we have the same issue with the ordinalness (sp?) of the utility function.

Answer – by how much (by what percentage) must you increase consumption in the economy to in order to make a consumer indifferent between economy one & economy two?

Find Δ such that $u(c_1) = u(c_2(1+\Delta))$

$$\Delta = \frac{c_1 - c_2}{c_2} = \frac{c_1}{c_2} - 1 \qquad simple, basic, easy, \blacksquare$$

You can expect this on a problem set

Now the challenge will be going to an infinite sum, continuous time….

Example – generalize to a dynamic economy with preferences,

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t)$$

An allocation is a sequence $\{c_t\}_{t=0}^{\infty}$

Economy 1 $\{c_{1t}\}$

Economy 2 $\{c_{2t}\}$

Given the above discussion, we are assuming that $U_1 > U_2$

Compute Δ such that $U_1 = \sum_{t=0}^{\infty} \beta^t u\big(c_2 + (1+\Delta)\big)$

Where Δ is the percent that consumption in Economy Two must increase **at each date** so the consumer is indifferent between economies.

Other Interesting Welfare Calculations

1) Steady State – Cross Steady State Comparison.
   Suppose c_1^* & c_2^* are the SS levels of consumption, then

$$\Rightarrow \sum_{t=0}^{\infty} \beta^t u(c_1^*) = \sum_{t=0}^{\infty} \beta^t u\big(c_2^*(1+\Delta)\big)$$

$$\Rightarrow \qquad \Delta = \frac{c_1^*}{c_2^*} - 1$$

2) Suppose Government in an economy like E_1 imposes a tax like the one in E_2. What does the dynamic path to the new SS look like?

"Suppose economy 2 is in SS, if they adopt a policy of economy 1, how much would their welfare increase?"

Let $\hat{c}_{1t}$ be the allocation that results from starting with your initial capital k_2^* with tax τ_{1k} $\forall t$

Meaning you are starting at $E_2,$ at the SS-two level of capital, and you change your tax policy to the tax policy of economy one. Again, the question is how does this compare to if we did not change the tax policy?

$$\sum_{t=0}^{\infty} \beta^t u(\widehat{c_{1t}}) = \sum_{t=0}^{\infty} \beta^t u(c_2^*(1-\Delta))$$

This second part asks how much do you need to increase SS consumption in economy two for this date & all future dates to be as well off as without changing the policy. How much better or worse off are you?

This lecture completes the tools to make welfare comparisons (& do your homework). You'll usually see these types of consumption equivalence comparisons.

This lecture ends with a review of the Prescott paper.

Why Do Americans Work So Much More Than Europeans?
by Edward Prescott - July 2004 (13 pages)
PDF: http://sites.google.com/site/curtiskephart/econ205a-adv-macro-i

I'm skipping the notes for it.