November 2nd 2009 - Lecture Twelve

Picking up from our previous lecture on Prescott & Taxation. Introducing Growth latter in the lecture. Our talk on growth will last about a lecture and a half.

$$max \sum_{t=0}^{\infty} \beta^t (\log c_t + \alpha \log(100 - h_t)$$

$$s.t. \quad (1-\tau_c)c_t + (1-\tau_x)x_t = (1-\tau_h)w_t h_t + (1-\tau_k)(r_t k_t - \delta k_t) + T_t$$

Remember our old constraint with $c + x = wh + (1-\delta)rk$?, a simpler time.

FOCs

$$c_t: \quad \beta^t + \frac{1}{c_t} = \lambda(1-\tau_c)$$

$$h_t: \quad \alpha\left(\frac{\beta^t}{100-h_t}\right) = \lambda(1-\tau_h)w_t$$

$$\Rightarrow \frac{\alpha c_t}{100-h_t} = (1-\tau)w_t \qquad where\ 1-\tau = \frac{1-\tau_h}{1-\tau_c} \quad \rightarrow \quad \tau = \frac{\tau_c - \tau_h}{1-\tau_c}$$

Why can we do this? τ_c & τ_h are very much linked. No one works (h) for the joy of working, they work for the things they can buy. A tax on consumption makes every dollar you earn less valuable to you, you thus want to include that in your labor market decisions. The distortions of taxes on consumption and labor enter into our model in a symmetric manner.

Firm's Solve

$$\max_{k_t, h_t} A_t k_t^{\Theta} h_t^{1-\Theta} - r_t k_t - w_t h_t$$

FOC

$$h_t: \quad w_t = (1-\Theta)A_t k_t^{\Theta} h_t^{1-\Theta} \quad = (1-\Theta)\left(\frac{y_t}{h_t}\right)$$

With these two first order conditions we can substitute out the wage and solve for h_t

$$h_t = \frac{\alpha c_t}{1-h_t} = (1-\tau)(1-\Theta)\left(\frac{y_t}{h_t}\right) \qquad 1 \rightarrow 100\ for\ ease\ of\ writting$$

$$\frac{\alpha c_t h_t}{y_t} = (1-\tau)(1-\Theta)(1-h_t)$$

$$\left[(1-\tau)(1-\Theta) + \frac{\alpha c_t}{y_t}\right] h_t = (1-\tau)(1-\Theta)$$

$$h_t = \frac{(1-\tau)(1-\Theta)}{(1-\tau)(1-\Theta) + \frac{\alpha c_t}{y_t}}$$

Now that we have an expression for h_t, how do we fit the model to the data for the United States? We call this process calibrating. We need to calculate numbers for our parameters.

Θ is a standard parameter. Theta is the capital share in the production function. The share of returns that go to capital. If you solve this problem for the wage/labor is paid (1-Θ) $* \textit{the Marginal Product of Labor}$ and capital is paid Θ.

If you go across countries you can measure what share goes to labor and capital as a share of output – and actually put a number to this parameter. Things vary a bit, but generally:

Capital Share $\boldsymbol{\Theta}$ – the share of returns that goes to capital is usually about a third
Labor Share $(\mathbf{1}-\boldsymbol{\Theta})$ – the share of returns that goes to labor is usually one-third.

Prescott: $\Theta = 0.32$

Alpha – α – Prescott picked alpha so that aggregate hours worked in the United States for the tax rates that he chose, leaves $h_t = 20\%$ of time worked (20 out of the 100 hours of work in a week, or whatever our data gives us). That makes

$\alpha = 1.54.$

With this simple model, Prescott finds these two parameters and compares labor data across about four countries during two four-year periods of times and with that choice feels that he can reconcile the a fair amount of data - the differences in weekly hours worked – Prescott's claim is that taxes are able to account for a fairly large change in hours worked (see table in paper to see)

Alpha – $\boldsymbol{\alpha}$ – a preference parameter. It relates to, 'how does the disutility of labor (or the utility of leisure) compare to the utility of consumption?' We can ask the question is 1.54 a reasonable number for this parameter?

Higher values of Alpha mean that your utility response to changes in h are bigger.

How can we be sure of our value for Alpha? – Micro Economists will run survey's of people's behavior to estimate labor supply elasticity. Testing how much an individual's work effort change given a percent change in wage.

Frisch Labor Supply Elasticity

$$\frac{\textit{change in hours worked}}{\textit{change in wage}}, \textit{holding wealth constant}$$

The wealth constant labor supply elasticity.

How can we calculate that in this model? Go back to h_t *formula*. Now solve it with the wage in.

$$h_t = 1 - \frac{\alpha}{(1-\tau)w_t}$$

Taking the derivative of h with respect to w

$$\frac{dh_t}{dw_t} = \alpha \frac{1}{(1-\tau)w_t^2} = \frac{1-h_t}{w_t} \ (\textit{from our FOC})$$

This is derivative, we want the elasticity, thus,

$$\textit{Elasticity} = \frac{dh}{dw} \cdot \frac{w}{h}$$

$$\frac{d\log h}{d\log w} = \frac{dh}{dw}\cdot\frac{w}{h} = \frac{1-h}{h}$$

This works to be about 3. – This calculation depends on the number of hours available in the week (if I doubled the amount of hours and h was still 20, then it's going to change how elastic things are without changing what the underlying economics mean – just keep that in mind). Thus two things are going into this calculation, one if the parameter Alpha and two is the normalization of the hours available in the week.

Point – this number -3- is higher than people normally estimate. If you look at micro estimates of this elasticity people normally get between 1/5 & 2. Two would be on the high end of most of these studies. If you plug in a lower value *here* then taxes are going to explain less of the changes in hours worked that we see. Thus Prescott's argument hinges on this elasticity of 3 – the higher the elasticity the more labor changes the higher you tax it.

There is an on going literature (of which Gorry enjoys) that questions the methods that micro studies use to estimate the parameters. And questions if those methods are fair at aggregating individual behavior. – Basically, this number if controversial, which makes the results that Prescott find –that taxes can explain a lot of the variation in hours worked found over time and between countries – also controversial.

Sounding like a drug dealer – Gorry, who specializes in labor economics, ends with "if you're interested in more, and want to talk about this…."

Economic Growth

There is a large amount of literature on economic growth asking important questions like why some countries are so much richer than others, trying to explain patterns of growth that have been observed. The topic is also popular amount many phd students at SC. These lectures will loosely starch the surface of that literature. Hopefully giving us tools to delve in. Tools that allow you to introduce economic growth into the model.

If Economic Growth is a topic you're interested in, these classes will hopefully help you understand the papers and the literature out there – get you started on all the reading you'll need to do later. A little heads up on how it works to be a grad student – you'll have to do all the research and reading on your own in later years. But we can do these classes together to help you get started.

Be sure to read:
On The Mechanics of Economic Development
by Robert Lucas, Feb 1988
PDF at http://sites.google.com/site/curtiskephart/econ205a-adv-macro-i

Kaldor's Growth Facts – people have been recording economic activity for a long time. And the list of facts that are pretty consistent are call Kaldor's -- stylized facts of economic growth. (a stylized fact is a simplified presentation of an empirical finding)

- Growth in output per person is constant. (about 1.5-2% a year for past 100 years.
- Ratios of $\frac{K}{Y}, \frac{I}{Y}, \frac{C}{Y}$ are constant
- Real interest rates are constant

later you'll discover this requires labor augmenting technology.

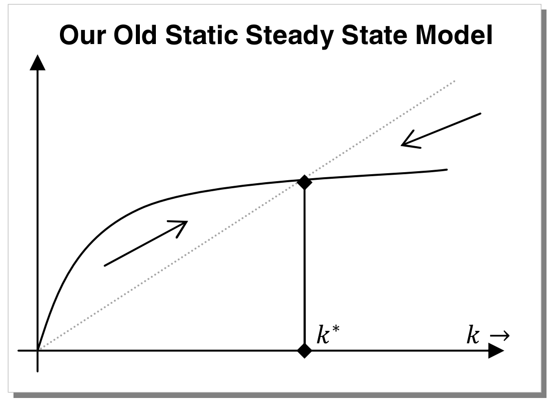
- Real wage grows at same rate as output per person – Y/N

later you'll discover this requires labor augmenting technology.

- Time working is constant
- Labor & Capital shares are constant

Obviously some things change over time. But fluctuations have been around some range of consistent levels.

So far our models have had no long term growth – only a steady state. Such models won't tell you how to generate long-term growth, let alone growth that isn't toward the one steady state.



Question - how do you add long run growth to our model?

$$y_t = F(k_t, h_t) = A_t k_t^{\alpha} h_t^{(1-\alpha)}$$

We now add growth to A_t Where before we assumed A was constant over time, now that is not the case.

1) Neutral Technological Change

$$A_t F(k_t, h_t) = A_t k_t^{\alpha} h_t^{(1-\alpha)}$$

2) Capital Augmenting Technology

$$F(A_t, k_t, h_t) = (A_t k_t)^{\alpha} h_t^{1-\alpha} = A_t^{\alpha} k_t^{\alpha} h_t^{(1-\alpha)}$$

3) Labour Augmenting Technology

$$F(A_t, k_t, h_t) = {k_t}^{\alpha} (A_t h_t)^{1-\alpha} = A_t^{1-\alpha} k_t^{\alpha} h_t^{(1-\alpha)}$$

I of course ask, couldn't you call capital a "technology that augments labor" already?

In order for two of the growth facts above to appear in our model, we need to have Labor Augmenting Technology. (1) the real interest rates are constant, & (2) the real wage grows at same rate as output per person – $\frac{Y}{N}$.

In general these two facts will only hold with LA-Technological Change ■. That technological change should show up in increasing wage rates **and not** in your returns to capital. (Somewhat important sidenote). Gorry: "what you need is that wage is increasing as a function of the technological change, meaning the real interest rates remain constant – that's going to work here because labor is going to be constant, while capital will grow. Capital will grow to drive (down) the marginal returns to capital while labor remaining constant is going to become more and more productive, earning a consistently higher wage."

$$y_t = A_t k_t^{\alpha} h_t^{1-\alpha}$$

$$\Rightarrow \log y = \log A + \alpha \log k + (1-\alpha) \log h$$

$$\frac{\dot{y}}{y} = \frac{\dot{A}}{A} + \alpha \frac{\dot{k}}{k} + (1-\alpha) \frac{\dot{h}}{h}$$

Another Important Feature
In order to get a balanced growth path we need preferences to be a certain way (this will be one of those problem set questions where we're given a lot of work to discover and prove this on our own).

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma} \qquad you'll\ need\ something\ like\ this.$$

These are the same preferences we've been using for some time. γ is an important parameter we'll learn more about in our Problem Sets.

Example of Economic Growth – A Quick Discrete Time Example
$h_t = 1,\ \ y_t = F(k_t, A_t, h_t) = k^\alpha (A_t h_t)^{1-\alpha}$

> Euler Equation in Standard Model is
> $$(LHS)\frac{u'(c)}{\beta u''(c)} = (RHS) F_k(k_{t+1}, A_{t+1}) + (1+\delta)$$

Now suppose that we grow at A constant rate g, so $A_{t+1} = (1-g)A_t$
We want to find a balanced growth path where k, c, x, & y all grow at constant rates – we call the balanced growth path.

Left Hand Side (LHS)
Let's start with the LHS, assuming balanced growth preferences

$$u'(c_t) = c_t^{-\gamma}$$

If c growth a rate g', then $c_{t+1} = c_t(1+g')$. Using those two equations we can cancel out c_t

$$LHS = \frac{1}{\beta(1+g')\gamma} = which\ is\ a\ constant\ growth\ rate$$

If c grows at a constant rate then LHS grows at a constant rate

Right Hand Side (RHS)
The RHS will also work out to be a constant. Rather, we now need to show that this grows at a constant rate.

$$RHS = \alpha\left(\frac{A_t}{k_t}\right)^{1-\alpha} + (1+\delta)$$

If k_t grows at a constant rate then LHS is a constant.

What about y_t?,

$$y_t = F(h_t, A_t) = k^\alpha A^{1-\alpha}$$

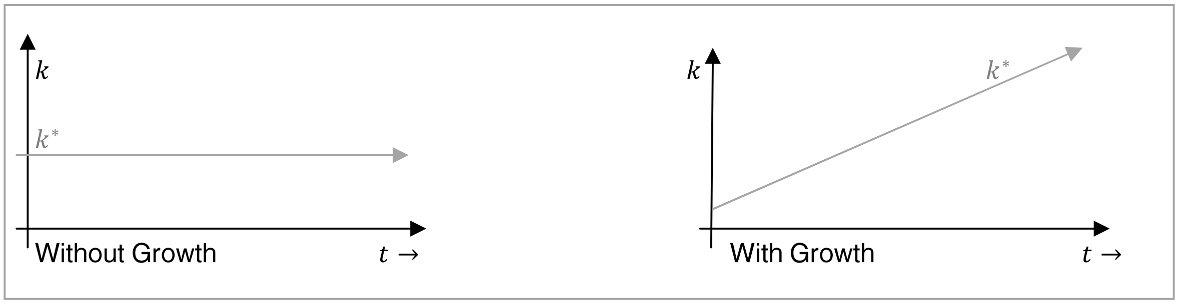
Meaning y_t grows at rate g

$$y_t = c_t + i_t$$

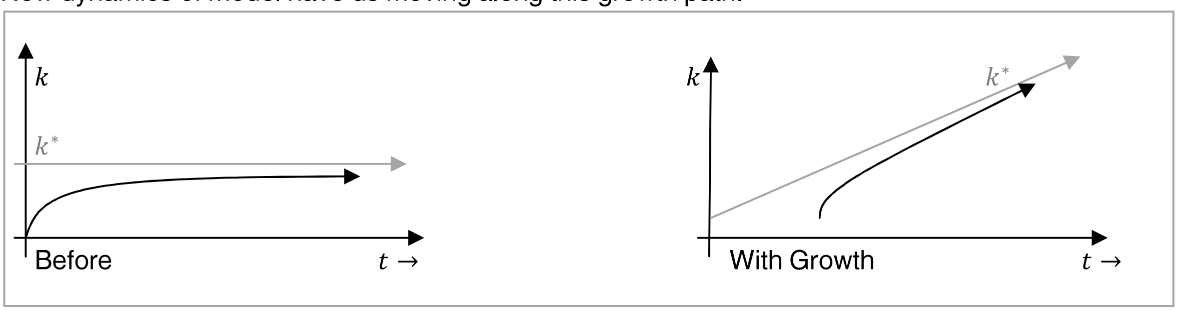
Everything Grows at g -- this is a particular result of having A as being labor augmenting.
Except $h_t = 1$

Now let's stop and think for a second. How do we think about how perfect all this is working out?

Adding Growth



Now dynamics of model have us moving along this growth path.



What is unsatisfying? A_t What is that? What is causing it to grow? We haven't really touched on that at all.

Many Countries growth over the years have looked like the following, with the US and UK and a select few others growing at constant rates – and many others not so consistent.