November 4th 2009 – Lecture Thirteen

Econ205A Macro Lecture 2009 11 04 Wed

Talking about the Lucas Development Paper – 'fun with growth rates today'

Logistics on what we'll be covering Friday with a Metrics Mid-Term due a few hours after.

Why read Lucas? "Lucas rarely writes but his papers good are filled with ideas." This paper is no different It presents a simple model of econ growth, discussing the benefits of that model by applying it to data, and then go through a few extensions of the model. Summary of a branch of research, & extensions.

We'll be going over the model he talks in his paper. Then we'l lget a problem set with one or more of the extensons he discusses.

Trying to understand econ growth
1)With growing population
2)Growth in tech – total factor productivity.
(10:00,pt1)

Lucas Growth Model
Pop Size $N(t)$
Assume $N(t)$ grows at rate θ
$N(t) = N(0)e^{\theta t}$ (15:15 pt1)

How to take growth rates in con't time model (12:00 pt1)

$$\frac{d}{dt}\log N(t) = \frac{\dot{N}}{N} = \theta$$

Preferences given by (13:00 pt discussed, how to interpet)

$$\int_0^\infty e^{-\rho t}\,\frac{(c^{1-\sigma}-1)}{1-\sigma}N(t)dt$$

We can interpet this as…..
N is amount of labor – thus prefs are proportional to the number of people in the econ – the amount of labor.
This model we have a population that is a mass that grows over time.

Two reasons why capital stock will grow (17:10 pt1)
1)
2) more and more people, labor force is getting larger.
But if you look at the per capita levels, you'll see the SS. But the aggregate capital level grows with population.

c is per capita consumption – thus total util is $c * pop$

You can also have one representative consumer – a different model.

Technollogy (21:00 pt1)

$y(t) = F\big(K,(t),N(t)\big)$
$A(t)k(t)^{\backslash alaph}N(t)^{1-\alpha}$
$\dot{K}(t) = X(t)$
$c(t)N(t) + X(t) = Y(t)$

$A(t)\ grows\ at\ rate\ \mu$

Combinging these eqs
$\dot{K}(t) = AK^{\alpha}N^{1-\alpha} - cN$
24:30pt1
Solving it with a Hamiltonian

$$\mathcal{H} = \frac{c^{1-\sigma}-1}{1-\sigma}N + \lambda[AK^{\alpha}N^{1-\alpha} - cN]$$

FOCs
c: $c^{-\sigma}N = \lambda N$
$\dot{\lambda} = \rho\lambda - \lambda[\alpha Ak^{(\alpha-1)}N^{1-\alpha}]$

Combining these two
$\Rightarrow -\sigma\left(\frac{\dot{c}}{c}\right) = \frac{\dot{\lambda}}{\lambda} = \rho - \alpha AK^{\alpha-1}\,N^{1-\alpha}$
28:15pt intuition….

We want to solve for all the growth rates of all the variables in the economy. A steady state will not be a solution if our N (and thus A) is growing. For a SS, you'd need K shrinking….

We are looking for a balanced growth path – a solution where all the vavriables grow at constant rates. We want to find the growth rate of c, k those will determine the growth rate of x and y, others (30:20pt1)

Let's start with
$-\sigma\left(\frac{\dot{c}}{c}\right) = \frac{\dot{\lambda}}{\lambda} = \rho - \alpha AK^{\alpha-1}\,N^{1-\alpha}$
Let's assume that $\frac{\dot{c}}{c}\,is\ a\ constant$ that c grows at a constant rate 31:45pt1

"Assume c grows at rate $\boldsymbol{\varphi}$"
$\rho + \sigma\varphi = \alpha AK^{\alpha-1}\,N^{1-\alpha} = "a\ constant"$
33:55pt 1 – how to approach this….

$\Rightarrow log\alpha + \log A + (\alpha-1)\log K = \log(constant)$
$\Rightarrow \frac{\dot{A}}{A} = (\alpha-1) * \frac{\dot{K}}{K} + (1-\alpha) * \frac{\dot{N}}{N} = 0$
$\Rightarrow \frac{\dot{K}}{K} = \frac{(\alpha-1)\theta - \mu}{\alpha-1}$
$\Rightarrow = \theta + \left(\frac{\mu}{1-\alpha}\right)$

36:30pt 1

$\frac{cN}{K} = \frac{\dot{K}}{K} = AK^{\alpha-1}N^{1-\alpha}$

Devivde all by K

$\Rightarrow \frac{cN}{K} = AK^{\alpha-1}N^{1-\alpha} - \frac{\dot{K}}{K}$

$there\ are\ a\ bunch\ of\ constants\ now \ldots 38:30pt1$

$\log c + \log N - \log K = \log(constant)$

$\frac{\dot{c}}{c} = \frac{\dot{N}}{N} - \frac{\dot{K}}{K} = 0$

$\frac{\dot{c}}{c} = \theta = \frac{\mu}{1-\alpha} - \theta$

$= \frac{\mu}{1-\alpha}$

40:40pt 1 intution on what all this means.

For practice, Let's solve fo the growth rate of output….

$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \frac{\alpha\dot{K}}{K} + (1-\alpha) * \frac{\dot{N}}{N}$

$= \mu + \alpha(\theta + \left(\frac{\mu}{(1-\alpha)}\right) + (1-\alpha)\theta = \mu + \frac{\alpha\mu}{1-\alpha} + \theta$

$\Rightarrow \frac{\mu}{1-\alpha} + \theta$

$44:20pt\ 1 - what\ this\ means, the\ growth\ rate\ of\ output\ is\ equall\ to \ldots$

What are the incentives for the people in the model. (45:00pt) They will accumulate capital as fast as the marginal production capital is increasing.

A is increasing…

So growth rate of aggregate output is a function of $\alpha, \mu\ and\ \theta -$ and what do each of these reflect?

$\frac{Y}{N} = \frac{\dot{Y}}{Y} - \frac{\dot{N}}{N}$

Compare to Reality.

What are observed growth rates of pop, output and capital.

…44:30pt1

Nope, labor and cap don't grow at same rate - . Also see labor shares- these have changed over time and across contries. Also included woman in the work force. This model does not capture those changes over time.

Growth effects vs Level effects

A common mistake is confusing policy changes that affect the level of capital and consumption, and those changes that affect growth-rates of capital and consumption .

52:00PT1

In these models the balanced growth model is like the ss level…. 2:45pt2

End of lecture on growth – just a taste of growth economics.

Outline – the next two or three lectures

Dynamic Programming

You can read of this at Chap 4-6 of Stokey Lucas Prescott Book. – 4 is preliminaries, 5 are examples and 6 is extentions.
4 and 6 of Leidquest and Sarget

Startnig out with non stochastic (non random) formulations.
We'll need some stocastics do to labor search, so we'll be learning that too.
These are basic twists to the growth model.

Consider the social planner prob

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$s.t.\ \ o \le c_t \le f(k_t) + (1-\delta)k_t$$

$l.o.m. of\ capital, k_{t+1} = f(k_t) + (1-\delta)k_t - c_t$
$with\ k_0\ given$

1) Solve for the optimal sequences of consumption & capital, $\{c_t\},\{k_t\}, starting\ with\ k_0$

2) You can also solve the problem recursively.

Motivation:
Suppose $\{k_t^*\}$ solves the problem. Then what will be optimal if you started with k_1^*
Then solution is k_2^*, k_3^* .... the titme period is unimportant. It's all about the level of capital. What maters if the level of capital you're at going forward. (16:00 pt2)

You want to solve a function for each vavlue of capital. For any level of capital today, you know what to do and you know what to do in future.

$if\ k = k'in\ period\ t$
$then\ the\ future\ sequence\ of\ k\ does\ not\ depend\ on\ t$ ...
(18:00pt2)

$we\ want\ to\ solve\ for\ an\ optimal\ policy\ function\ g(k)$
$given\ g, \quad k_1^* = g(k_0),\ \ k_2^* = g(k_1^*)$ ...
If you know g, you know the solution to any k you are given. You're solving the model for any given value of k.

Example – a finite horizon model.
(21:00pt 2 what s and a stand for…)

$$\max_{\{s_t\},\{a_t\}} \sum_{t=0}^{\infty} \beta^t u(a_t, s_t)$$
$$s.t. \quad \psi(s_t, a_t) \ge 0$$
$$s_{t+1} = h(s_t, a_t)$$
$$s_0\ given$$

Different g, just our feasibility constraint.

1) $choice\ problem\ with$

$2(T+1)-1$ *variables*
and $2(T+1)$ *constraints*

2) Alternately, you can think of contingent plans. Start at end of problem (in period t=T), and solve backwards. (joke at 27:00 pt2

suppose at date $T, s_t = s$

That means you can solve….

$$\max_{a_t} u(a_t, s)$$
$$s.t.\quad \psi(s_1, a_T) \geq 0$$

Let $\pi_0(s)$ *be the optimal choice function.*

$a_T^* = \pi_0(s)$

Define the value you attain by entering the …30:45pt2

$$V_0(s) = u(\pi_0(s), s)$$

V_0 is solution at t=T. zero periods less.
The highest utility attainable, from period T *on, assuming you entered period* T *with* $s_T = s$

next, assuming you enter $s_{T-1} = s$
what is the optimal behavior from this point on?

we need to choose a_{T-1} *and* a_T *to maximize utility*

Claim: we can reduce the problem to choosing a_{T-1}.

$$V_1(s) = \max_{a_1, s'} u(a,s) + \beta \quad V_0(s')$$
$$s.t.\quad \psi(s,a) \geq 0, \quad \& s' = h(s,a)$$

$\Pi_1(s) = a$ *be optimal choice function*
s' is the next period value of s

And we can keep doing this period by period until we reach our given k_0

$$V_T(s) = \max_{a,s} u(a,s) + \beta V_{T-1}(s')$$
$$s.t.\quad \psi(s,a) \geq 0$$
$$h(s,a) = s'$$
$$\Pi_T(s)$$

We have a sequence of $T+1$ *one dimensional optimization problems.*
40:50pt2 = notes on what this is doing….
This is all motivation for setting up the general framework to solve problems recursively.