November 13th 2009 - Lecture Fifteen

Econ205A Macro – 20091113 Friday

Outline – Going over mid-term and problem problems. Mid-Term Question One (also PS5 Question One). Plus all of Problem Set Six.

Review of Mid-Term Question One - aka Problem Set Five Question One Solution

Part 1) Write down a problem that defines the set of efficient allocations.

Grading this, if you could write down the maximization problem down with all the equations there you'll get full credit. (Gorry thinks everyone got full credit.

An ideal solution would be to do all the simplifications and to write down the problem you plan to solve. So, a probl that leads to only one Lagrange multiplier

Gorry's Answer: substituting out for consumption. Substituting out for h_{ct}. This is a social planner's problem. For the feasibility constraint, you want to substitute all you can so you just have it in terms of $\mathrm{k_{ct}}, \mathrm{k_{it}}, \mathrm{h_{it}}$ and k_{ct+1}& k_{it+1}

$$\max_{\mathrm{k_{ct}},\mathrm{k_{it}},\mathrm{h_{it}}} \sum_{t=0}^{\infty} \beta^t\, u\big(F(k_{ct}, 1-h_{it})\big)$$
$$s.t.\ \ k_{ct+1} + k_{it+1} = (1-\delta)(k_{ct}+k_{it}) + G(k_{it}, h_{it})$$

Part 2) Derive first order conditions that characterize an efficient allocation.

Gorry's method skips the Lagranian,

Seeking them with respect to, k_{ct}, k_{it}, h_{it}, and let λ_t *be* the multiplier on the constraint

You're getting k_{ct} in two different time periods.

$$(1) k_{ct}:\ \ \beta^t u'\big(F(k_{ct}, 1-h_{it})\big) F_k\big(F(k_{ct}, 1-h_{it})\big) = \lambda_t(1-\delta) - \lambda_{t-1}$$

$$(2) k_{it}:\ \ \lambda_t[(1-\delta) + G_k(k_{it}, h_{it})] = \lambda_{t-1}$$

$$(3) h_{it}:\ \ -\beta^t u'\big(F(k_{ct}, 1-h_{it})\big) F_h(k_{ct}, 1-h_{it}) = \lambda_t G_h(k_{it}, h_{it})$$

Simplifying

$$(1) \& \ (2): \beta^t u_t' \, F_{kt} = -\lambda_t G_{kt}$$
$$\Rightarrow \frac{F_{kt}}{F_{ht}} = \frac{G_{kt}}{G_{ht}} \quad \text{the static condition} \quad ■$$

It says the ratio of the MP of capital to the MP of labor in each period must equal production function must be the same.

Using Equation (3), we can do a standard time shift (back one period) to get a λ_{t-1}/λ

$$\Rightarrow \frac{\beta^{t-1} u_{t-1}' F_{ht-1}}{\beta^t u_t' F_{ht}} = \frac{\lambda_{t-1} G_{ht-1}}{\lambda_t G_{ht}}$$

Using the λ_{t-1}/λ from (2) again, gives,

$$\frac{u_{t-1}' F_{ht-1}}{\beta u_t' F_{ht}} = \frac{G_{ht-1}}{G_{ht}}[(1-\delta) + G_{kt}] \quad ■$$

■ These are the two conditions we were looking for in part II

Part 3) You're asked to do two things. First, define a SS for the economy. Then impose SS and find equations that determine levels of k_{ct}, k_{it}, h_{it}

- "A steady state is a value k_0^*, s.t. if $k_0 = k^*$, then $k_t = k^* \; \forall t$."

Memorize the above incantation for future midterms, finals and prelims.
That is, we are looking for three equations in k_{ct}, k_{it}, h_{it}

From the static condition, we can change it to

$$\frac{F_k(k_t^*, 1 - h_i^*)}{F_h(k_t^*, 1 - h_i^*)} = \frac{G_k(k_t^*, 1 - h_i^*)}{G_h(k_t^*, 1 - h_i^*)}$$

From the second equation

$$G_k(k_t^*, 1 - h_i^*) = \frac{1}{\beta} - (1 - \delta)$$

The 'third equation' is just the feasibility condition.

$$\delta(k_t^* + k_c^*) = G(k_t^*, 1 - h_i^*)$$

This says that the production of capital goods in each period is just enough to control for the depreciation.

For this problem, although the algebra gets a little bit tedious with two production functions – time pressure. Solving problems like this should be strait forward.

Part 4 however requires you to think about setting up a model in a different situation. Hard on a test. Many missed this.

Part 4) QUESTION: *In the above parts we have assumed that the capital used to produce consumption and investment are the same. Suppose that you wanted to assume that capital used to produce consumption and investment goods are different, but that a unit of the investment good can be used as either form of investment. Modify the equations describing the technology to describe this new economy. You do not have to solve this problem!*

Where before capital produced in the production function could be used to produce either type of good and can be changed freely period to period – we now want to say that once you've created a certain type of capital it's fixed, and in each period you cannot reallocate your capital between capital used to create consumption and capital used to create investments (note, you'll see a problem very similar to this in dynamic programming, problem set seven)

You're just asked to write down equations of the model. Consumption is produced using the production function, with capital for just consumption. Investment has its own production function with capital only used for investment production. That implies that we need to split investment between the two types of capital. And we need laws of motion for the two types of capitals.

$$c_t = F(k_{ct}. h_{it})$$
$$i_t = G(k_{it}, h_{it})$$
$$i_t = i_{ct} + i_{it}$$
$$k_{ct+1} = (1 - \delta^c)k_{ct} + i_{ct}$$
$$k_{it+1} = (1 - \delta^i)k_{it} + i_{it}$$
$$h_t = h_{it} + h_{ct}$$

Problem Set Review 6 – Part One.

See Problem Set. In part two I've included the class notes into my group's answer.

Problem Set 6 – Part II

See updated Problem Set