November 30th 2009 - Lecture Twenty

Econ205A Macro – 20091130 – Outline – Continuing Notes on Dynamic Programming in Stochastic Setting. The Stochastic Growth Model - A Recursive Competitive Equilibrium for the Stochastic Growth Model. Labour Search – introducing a simple labor search model.

The last homework assignment has us set up Bellman Equations with uncertainty.

Outline – going into the technical details of dynamic programming in stochastic setting.

For Final – more interested in setting up equation and solving the problem than memorizing every technical detail. Although our lectures (particularly the past several) have gone into great technical detail, you can expect the final to be more practical.

Continuing Notes on Dynamic Programming in Stochastic Setting.

One restriction that we've placed so far is that we are choosing y next period. And the random shock gets realized. One thing that might be the case is that one's state next period could also depend on the shock next period. So if you choose y it enters your objective function, but it doesn't go directly into the state next period. That if you state next period in this alternate set-up depends on both your choice of y and what shock is realized next period.

Gorry will show to us that that is actually not a problem. We can just define a law of motion that depends on the shock. The law of motion will just get plugged into the value function.

Define a Law of Motion for next period's state

$$x' = \varphi(x, c, z')$$

(6:00)

$$\Rightarrow \quad V(x,z) = \max \left[F(x,c,z) + \beta \int V(\varphi(x,c,z'), z')\ q(z,z')\ dz'\right]$$

(6:33) on integral

Summary of second term…(9:44)
There are a number of way to write this

We need a c for choice variable because, where we had y before, now the state tomorrow x' is also a function of z'.

Summary of Main Result

1) If $F, \beta, \& \Gamma$ are as before (12:20)
   And q satisfies required properties, then V^* is the unique solution to the Bellman Equation.
   And V^* is bounded and continuous (bounded on the state space).

We need the value function to be increasing in x, thus

2) If $F\ \&\ \Gamma$ are increasing as before, and φ is increasing in x, then V is increasing in x.

convexity

3) If the graph of Γ is convex and $F(\cdot,\cdot,z)$ is concave for each z, and $\varphi(\cdot,\cdot,z')$ is concave for each z'.
   Then V is concave in x. And g is going to be single value and continuous.

(16:45 on $F(\cdot,\cdot,z)$) (18:30 on g)

Stochastic Growth Model -
Let's do one set up of the growth model with stochastic productivity

Preferences are standard. New thing (20:55) are the shocks….
Preferences:

$$E\left[\sum_{t=0}^{\infty} \beta^t \, u(c_t, \;\; 1-h_t)\right]$$

$$tech\colon \; y_t = z_t F(k_t, h_t)$$

Tech: 22:15. It can be many things

Markets Clear

$$c_t + i_t \le y_t$$

$$k_{t+1} = (1-\delta)k_t + i_t$$

Law of motion for z_t

$$\{z_{t+1}\} \; described \; by \; Q(z_t, z_{t+1})$$

By zt, be means whatevery zt is it's described by … (29:25)

$$k_0 \; given$$

Final Exam Question (28:45)
Using this, set up Bellman Equation for Social Planner Problem.

Step One – define variables
State Variables – $z \;\&\; k$
Choice Variables – c_t, h_t, k_{t+1}
You could sub out c_t, but for this we won't.

32:00 – missed what belongs after the max $u(c_t, \;\; 1-h_t)$

33:00 knowing $z_t, this\ is$ the expectation that.

$$V(z_t, k_t) = \max_{c_t, h_t, k_{t+1}} \left[u(c_t, \;\; 1-h_t) + \beta E_{z_t}[V(z_{t+1}, k_{t+1})]\right]$$

$$s.t. \quad k_{t+1} = z_t F(k_t, h_t) + (1-\delta)k_t - c_t$$

$$E_{z_t}[V(z_{t+1}, k_{t+1})] = \int_Z V(z_{t+1}, k_{t+1}) \;\; Q(z_t, z_{t+1}) \, dz_{t+1}$$

$$= \sum_{z=1}^{Z} \Pi_{ij} \dots$$

There are all kinds of ways to set this part up

For Practice – Let's write down the Competitive Equilibrium for this Problem
A Recursive Competitive Equilibrium for the Stochastic Growth Model **(see 11/17)**
Is a list of functions $V(k,K,z),\ k'(k,K,z), K'(K,z), h(k,K,z),\ H(K,z),\ r(K,z), w(K,z)$

$h(k,K,z)$ at 41:30 ish-explained what it is why we want it. It wasn' t in11/17 lecture. Only state variable can go into those functions

$s.t.$
1) Consumers Maximize – taking prices $(r(K,z), w(K,z))$ & aggregate decisions $(K'(K,z), H(K,z))$ as given $V(k,K,z)$, $k'(k,K,z)$ & $h(k,K,z)$ solve

(46:15 on wording)

$$V(k,K,z) = \max_{\bar{k},\bar{h}} u(w(K,z)\bar{h} + r(K,z)k - \bar{k},\ \ 1 - h_t) + \beta E_z\, V(\bar{k}, K'(K,z), z'$$

$$and\ \ k'(k,K,z),\ \ h(k,K,z)\ \ are\ optimal\ decision\ rules$$

$c + i = wh + rk$ – we want to solve for c
$c = wh + rk - i \rightarrow wh + rk - \bar{k} + (1-\delta)k$

$\bar{k}, \bar{h}$ - the bars are for those variables you choose – h, labor this period and k, capital next period

Compared to 11/17 – we substituted out c_t - thus no constraint for us

2) Firm Maximization
$\forall\ K, z$
$r(K,z) = z\,F_1\big(K, H(K,z)\big)$
$w(K,z) = z\,F_2\big(K, H(K,z)\big)$

3) Markets Clear
Nothing to say here, depends on the model

4) Consistency Condition
$\forall k, z$
$k'(K,K,z) = K'(K,z)$
$h(K,K,z) = H(K,z)$

This has been the general overview of stochastic models

Now – time for Labor Search

1:00:00 – Labor Search Model

Worker Search Model
We need to know the chances of them working – and distribution of pay might be if offered work

- Unemployed Worker
- Their are goal is to maximize their lifetime wealth
- Each period an unemployed worker gets one job offer

- Jobs/Wages are drawn from a known distribution $f(\cdot)\ with\ \ F(\cdot)\ cdf$

We need to know something about their skill set.

- When unemployed they receive some benefit b

b can be interpreted as the cash value of unemployment. Perhaps also your value of leisure plus the government money (or lack of money) you might get.

- β is the discount factor.

Assume the distribution of wages F has support on $[0, \bar{w}]$ (there is a probability of getting wage offers in this range) where $\bar{w} > b$

Given worker keeps job forever

Once a worker accepts a job they keep it forever. It makes the value function very simple.

At the beginning of each period you get a wage offer $\hat{w}$ – you can accept it (for all time), or wait, and keep searching.

$$V(\hat{w}) = \max\left\{\frac{\hat{w}}{1-\beta}, b + \beta \int_0^{\bar{w}} V(w')\, f(w')\, dw'\right\}$$

Two options, if you accept the job, or don't and wait next period

$f(w')$ is the probability of getting these draws

A thought about how to solve this model 1:27:20 – just plot both of those guys

The optimal value if associated with the upper contour

See vid for which line is which.