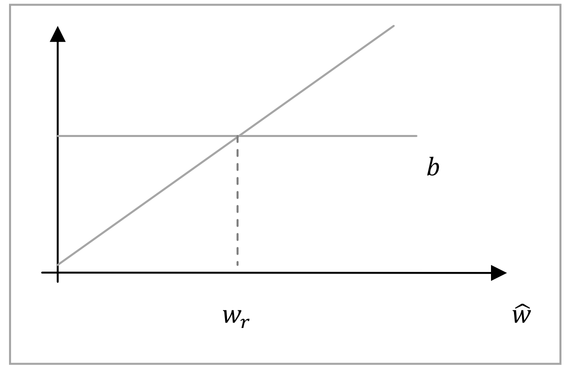
December 2nd 2009 - Lecture Twenty-One

Econ205A Macro - 20091202



From last time

$$V(\widehat{w}) = \max\left\{\frac{\widehat{w}}{1-\beta}, b + \beta\int_0^{\overline{w}} V(w')\, f(w')\, dw'\right\}$$

$$V(w_r) = \frac{w_r}{1-\beta} = b + \beta\left[\int_0^{w_r} \frac{w_r}{1-\beta}\, f(w')\, dw' + \int_{w_r}^{\overline{w}} \frac{w'}{(1-\beta)}\, f(w')\, dw'\right]$$

$$\left(\Rightarrow b + \beta\int \; V'(w')\; dw'\right)$$

$$= b + \beta\cdot\frac{w_r}{1-\beta}\, F(w_r) + \beta\int_{w_r}^{\overline{w}} \frac{w'}{1-\beta}\, f(w')\, dw'$$

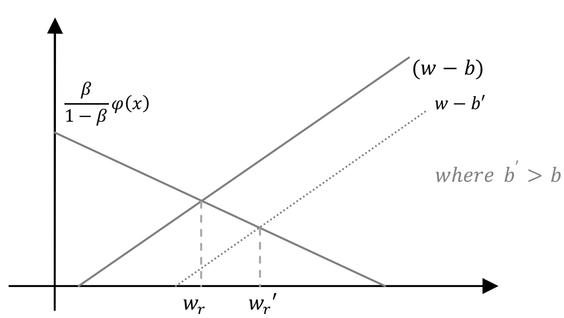
subtract $\beta\cdot\frac{w_r}{1-\beta}$

$$\Rightarrow\; w_r = b + \frac{\beta}{1-\beta}\,\varphi'(w_r)$$

$$\varphi(x) = \int_x^{\overline{w}} (w-x)\, f(w)\, dw$$

$\Rightarrow \quad \frac{\beta}{1-\beta}\varphi(x)$ *is your expected surplus of your potential to draw a higher wage*

Properties



$$\varphi'(x) = 0 - \int_x^{w} f(u)\, dw \qquad \varphi(0) = \textit{mean wage} \qquad \varphi(\overline{w}) = 0$$

1) example

worker loses job with some probability, lambda. Suppose a worker starts a period with job in hand and wage offer $\hat{w}$. They still have the same two choices. They can either continue working at the job with wage w\hat. Or switch at the new wage.

Now the worker loses job with probability lambda, and keeps the job with probability (1-\lambda).

- $\hat{w} + \beta\left[\lambda\left(b + \beta\int_0^{\bar{w}} V(w')\, f(w')\, dw'\right)\right]$ If you lost your job you get b (unemployment) plus the future value. If you
- $(1-\lambda)V(\hat{w})$ this is if you don't lose your job.
- $b + \beta\int_0^{\bar{w}} V(w')\, f(w')\, dw'$ lastly the alternative is that you just don't have a job.

$$V(\hat{w}) = \max\left\{\hat{w} + \beta\left[\lambda\left(b + \beta\int_0^{\bar{w}} V(w')\, f(w')\, dw'\right) + (1-\lambda)V(\hat{w})\right] \quad , \quad b + \beta\int_0^{\bar{w}} V(w')\, f(w')\, dw'\right\}$$

2) example extension – worker choice their search intensity. They can choose the probability at which they get a job offer each period – and there is a cost associated with that.

Worker Chooses Search Intensity π with cost $c(\pi)$

π is probability that worker gets a job offer each period.

Properties on c_t - $c(0) = 0$; $c'(0) = 0$

$\lim_{x\to 1} c'(x) = \infty$

$$V(\hat{w}) = \max_{\pi,\{a,r\}}\left\{\frac{\hat{w}}{1-\beta}, b - c(\pi) + \beta\left[\pi\int_0^{\bar{w}} V(w')\ f(w')\, dw' \ + (1-\pi)V(0)\right]\right\}$$

You can also define $V(0)$ as some other value function, $U = V(0) = \max_\pi b + c(\pi) + \beta[\pi\int \dots dw' + (1-\pi)u]$

Final – as long as the mid-term

Two problems -

He has written the final, written during the mid-term.

Set up problems and do basic types of things

****Part two audio started -

- Know how to set up a Lagranian/Hamiltonian, to get to focs and the euler equation.
- Given bellman – take focs and envelope and to get to euler eq
- Be able to set up dynamic programming stuff.
- The last classes have been a bit on growth & taxes but otherwise lots of dynamic programming stuff.
- Dynamic programming questions, identifying the state. Setting up bellman.
- Working through to find euler eq.
- Also setting up these types of labor problem

Homework – two types. One, challenging and hard, but you need to see. Two, just like what's on the Final.

Some algebra – no guessing and checking. No proofs on conditions for bellman to work

Set up, FOCs, …. Then analyze the model, or do something else.

Mid-Term example of Final – setting up model

PS7 – setting up Bellmans.

PS8 – lots of trouble with this, thus we'll do that in class.

PS 9 – Friday we'll go over those problems.

Problem Set 8 – (6:40)

First Problem – Human Capital Accumulation
Set it up as a sequence problem

Human capital is equal to

$$V^*(k) = \max_{\{k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \; u\left(f\left(k_t\, \phi\left(\frac{k_{t+1}}{k_t}\right)\right)\right)$$

$c_t = f(L_t)$
$L_t = k_t h_t$
$h_t = \phi\left(\frac{k_{t+1}}{k_t}\right)$

$s.t. \quad k_{t+1} \in [(1-\delta)k_t, (1+\lambda)k_t]$

$k_0 \; given$

Call; $(1-\delta)k_t \; = \Gamma(k)$

$$b)\; V(k) = \max_{k' \in \Gamma(k)} u\left(f\left(k\phi\left(\frac{k'}{k}\right)\right)\right) + \beta\, v(k')$$

11:45
$\beta(1+\lambda) < 1$

If we only produce human capital (HC), that implies
$\Rightarrow \;\; k \; grows \;\; (1+\lambda) \; each \; period$

$$\frac{k_{t+1}}{k_t} = (1+\lambda) \quad , \quad \phi(1+\lambda) = 1$$

$$\Rightarrow \beta^t\, U\left(f\left(k\phi\left(\frac{k'}{k}\right)\right)\right) \leq \beta^t\, U\big(f(k_0(1+\lambda)^t)\big)$$

14:30

$$< \beta^t (1+\lambda)^t\, k_0 = [\beta(1+\lambda)]^t k_0$$

c) Going through all of ht eproperties – don't want to cover much of that in a Final review, but he does want to cover
(22:10) good stuff
28:00

d) Steady States
take focs and envelope to get euler equation

foc wrt k'

$$(1) \quad U'(\cdot)f'(\cdot)\,\phi'\left(\frac{k'}{k}\right)+\beta v'(k')=0$$

Envelope condition – derivative with respect to k

$$v'(k)=U'(\cdot)f'(\cdot)\,\left[\phi\left(\frac{k'}{k}\right)-\phi'\left(\frac{k'}{k}\right)\frac{k'}{k}\right]$$

Let's shift this forward

$$(2) \quad v'(k')=U'\left(k\left(k'\phi\left(\frac{k''}{k'}\right)\right)\right)\,f'\left(k'\phi\left(\frac{k''}{k'}\right)\right)\left[\phi\left(\frac{k''}{k'}\right)-\phi'\left(\frac{k''}{k'}\right)\frac{k''}{k'}\right]$$

Combining (1) & (2)

$$0=U'(\cdot)f'(\cdot)\,\left[\phi\left(\frac{k'}{k}\right)-\phi'\left(\frac{k'}{k}\right)\frac{k'}{k}\right]+\beta\left\{U'\left(k\left(k'\phi\left(\frac{k''}{k'}\right)\right)\right)\,f'\left(k'\phi\left(\frac{k''}{k'}\right)\right)\left[\phi\left(\frac{k''}{k'}\right)-\phi'\left(\frac{k''}{k'}\right)\frac{k''}{k'}\right]\right\}$$

$$k=k'=k''$$

$$U'\left(f\left(\bar{k}\phi(1)\right)\right)f'\left(\bar{k}\phi(1)\right)\phi'(1)+\beta U'(\cdot)f'(\cdot)[\phi(1)-\phi'(1)]=0$$

$$\phi'(1)+\beta\phi(1)-\beta\phi'(1)=0$$

$$\frac{\phi'(1)}{\phi(1)}=-\frac{\beta}{1-\beta}$$

let β^* solve this equation.

38:00